Sample Study Guide – Cover Letter

The ACE manual was designed with the intent of clarifying complex text (and problems) with explanations in plain-English. This is accomplished via clear and concise summaries of each chapter, author's commentaries for the most difficult ("Greek") material, ACE original problems, and more in-depth explanations and answers to some of the problems that are in the book. The study guide has calculations and practice problems integrated with the outline to facilitate learning.

In this sample we provide a chapter from the manual, along with a page from the formula list.

We would also like to put in a plug for the ACE life and online seminars. This will be an excellent opportunity for you to enhance your understanding of the syllabus. We will walk through book examples, outline key material, walk through old exam problems, and present ACE SOA type problems. The seminar will combine similar texts so you can learn/review the syllabus in a reasonable amount of time.

Please contact us with any questions at customerservice@acemanuals.com

Key Rate Durations: Measures of Interest Rate Risks (LRM-117-14)

Key Concepts

- Traditional duration matching is flawed because it assumes parallel yield curve shifts
- Key rate durations will approximate the change in an asset or liability with non-parallel yield curve shifts
- Key rate durations can be used for all types of securities with interest rate risk

1) Introduction and Overview of Macaulay duration and modified duration
   a) The study note assumes the reader is familiar with traditional duration metrics, so I have briefly summarized that in this section
   b) $Mac\ Dur = \frac{PV(CF \times t)}{PV(CF)}$
      i) You can think of Macaulay duration as a weighted average t where it is being weighted by the PV of cash flows
   c) Modified Duration = Mac Dur / (1+i)
   d) $Mod\ Dur = \frac{PV(CF \times t)}{(1+i)PV(CF)}$
   e) % Δ Price of Bond = -ModDur * Δi
   f) Can also solve for the implied ModDur by looking at the price of a bond before and after an interest rate change
   g) The above methods assume:
      i) Parallel yield curve shifts
      ii) Small changes in i
   h) Definition of Key Rate Duration (KRD)
      i) KRD will split the above formula for modified duration into component pieces
      ii) The sum of KRD will equal the modified duration
      iii) KRD measures the change in the price due to a change in a "key rate"
   i) Advantages of KRD
      i) Identifies the price sensitivity to each point on the yield curve
      ii) KRD aggregates linearly
      iii) KRD is applicable over a wide range of yield curve movements
      iv) Can create a replicating portfolio to mimic an embedded option
      v) Can compare different types of products/structures (how does an embedded option compare to a bond?)
      vi) Portfolios immunized using key rates will withstand non-parallel yield curve shifts

2) Key Rate Durations
   a) KRDs are not a single measure like Modified duration (also called "effective duration")
   b) Key rate durations can be linearly interpolated to find the other durations (Exhibit 2)
   c) First step is to identify the key rates
   d) KRD is solved for by looking at the price of a security before and after a small interest rate change on ONE key rate
   e) $KRD(i) = \frac{P-P^*}{P \times \Delta i}$

i) Said in another way, you first find the percent change in price (P-P*) / P
ii) Then you divide by Δi
iii) Note, this is exactly like the traditional modified duration, but you are only changing one key rate, not the entire yield curve
iv) Repeat process for all key rates → the final result is a vector of KRD

f) When you sum up all of the key rate durations, you get modified duration
g) For zero coupon bonds with maturity (t), key rate duration(t) = modified duration

3) Numerical example

So far I have used "Macaulay duration", "Modified duration" and "Effective duration". Let me clear the air. Macaulay duration is the weighted average time until cash flows are received, weighted by the PV of cash flows. Modified duration is the percentage change in price for a unit change in yield. When yields are continuously-compounded Macaulay duration and modified duration are equal. I use "effective duration" and "modified duration" interchangeably.

This section of the text says that effective duration of a zero coupon bond is equal to the maturity. Keep in mind that is for continuously compounding only. When yields are periodically compounded that is not true.

I have examples at the end of the text to show how to calculate durations using both continuous compounding and periodic compounding.

a) This example uses two portfolios to illustrate the advantages of KRD
   i) Portfolio #1: $50 of 2-year zero coupon bonds and $50 of 30-year zero coupon bonds
   ii) Portfolio #2: $100 of 16-year zero coupon bonds
b) The modified durations of these portfolios are the same → but they have very different interest rate risks
c) The example uses three scenarios to illustrate interest rate risk
   i) Scenario #1: 10bps down at all key rates
   ii) Scenario #2: 2-year rate increases by 10bps and 30-year decreases by 10bps
   iii) Scenario #3: 2-year rate decreases by 10bps and 30-year increases by 10bps
d) Exhibit 4 shows the change in value of the two portfolio (different in scenario 1 and 2)

4) Key Rate Durations and Alternative Schemes
   a) Ways interest rates can move:
      i) Level → all rates rise or fall together
      ii) Steepness → short term rates and long term rates moving the opposite direction
      iii) Curvature → short term rates and long term rates moving in the same direction, but mid-term rates not moving
   b) When you are assessing how immunized a portfolio is, you need to consider all three types of movements
   c) Key rate duration can handle all three of these

5) The key between key rate durations and pricing models
   a) You can calculate KRD on any type of interest sensitive asset, given you have a pricing model
   b) KRD is a good way to investigate interest rate risk on options

6) Discussion about different types of assets

a) Zero coupon bonds
   i) KRD(i) = Modified Duration
   ii) All other time periods KRD = 0

b) Coupon Bonds
   i) Usually have significantly large amount of risk attributable to the maturity payment
   ii) Can assess interest rate risk by term by looking at KRD as a percentage of modified duration

c) Callable corporate bond
   i) The call option in a corporate bond will decrease the modified duration and change the pattern of KRD
   ii) The call features make the bond more sensitive to short term key rate changes
   iii) Lower risk attributable to the maturity payment

d) Callable bond with sinking fund
   i) A sinking fund is where there is a bucket of money that is set up to pay off the face amount at maturity.
   ii) Very similar to callable bond

Don't worry about doing any calculations in this section. The author has a pricing model for all of these assets and options.

e) European Call and Put Options
   i) This option has the following characteristics
      (1) Underlying security is 30-year bond
      (2) Strike price = par value
      (3) 10-year duration
   ii) As expected calls and puts are opposite of one another
   iii) European call has a modified duration of 70.24! (this means it is very sensitive to changes in interest rates)

f) American Call and Put Options → since the option can be exercised at any time, duration is lower

g) Embedded option in a callable bond
   i) Similar to a non-callable bond + American option
   ii) So the embedded option in a callable bond analogous to an American option

h) GNMA Pass-Through
   i) Overview → mortgages are securitized and the cash flows are packaged into securities that resemble bonds, these are called mortgaged backed securities or MBS. The cash flows can be Principal and Interest, Principal only, or Interest only
   ii) KRD for GNMA pass-through is most sensitive to medium term yield curve

i) PO GNMA
   i) The PO has negative KRD up to the 7^{th} year and positive after that.
   ii) Think about what happens when there is a drop in interest rates → everyone wants to refinance
      (1) This leads to getting more PO cash flows
      (2) PO's are very sensitive to long term rates decreasing
      (3) This sensitivity is one sided → the same high sensitivity is not true when interest rates rise

j) IO GNMA
   i) Opposite is true for IO → when long term rates decrease, everyone refinances and the securities have no interest to pay! So the value of IO goes down as long term rates decrease.

7) Key rate durations in a portfolio context
   a) The key rate durations of a portfolio are the weighted sum of the key rate durations of each bond position
   b) D(i) = $w(1) \times D(1, i) + w(2) \times D(2, i) + \cdots + w(m) \times D(m, i)$
   c) Said in words, you take the weighted average of all security's key rate durations to get the portfolio key rate duration

8) Identifying interest rate bets using key rate durations
   a) Each key rate duration contributes to the total risk exposure
   b) If you are trying to do ALM, the goal should be to have all KRD match between assets and liabilities
   c) Example of trying to match an index
      i) The study note presents an illustrative example of a portfolio manager trying to mimic an index of fixed income securities
      ii) To accomplish this, the manager should build a portfolio where the KRD match the index
      iii) Key rate durations and total modified duration are given below:

Term	KRD
0.25	0.02
1	0.19
2	0.35
3	0.56
5	0.69
7	0.74
10	0.65
15	0.61
20	0.34
25	0.28
30	0.16
Mod Dur	4.59

      iv) The text says that a portfolio is constructed to have all key rate durations = 0.42
         (1) This portfolio would be immunized to level interest rate changes, but not steepness or curvature
   d) Estimating % changed given a yield curve shift
      i) % Change = Σ (Δi(t) * KRD(t))
      ii) The text does this formula for level, steepness, and curvature
      iii) "1.395" means the index value will change by 1.395% for the level yield curve change

9) Constructing hedge ratios for option embedded bonds
   a) You can construct a replicating portfolio of zero coupon bonds to mimic any interest sensitive financial instrument
   b) Process → find the weights for each zero coupon bond and then solve for the amount of cash to hold
   c) W(i) = D(i) / T(i)
      i) D(i) = Key rate duration for point on the yield curve i
      ii) Term of point "i" on the yield curve (so if point 11 is a 30-year bond, then T=30)
   d) W(0) = amount of cash to hold, this is solved for so the sum of weights equals 1.00
   e) Text has an example to illustrate these formula

10) Appendix: Adjusted Key Rate Durations
   a) This expands key rate durations to more than just spot rates and zero coupon bonds
   b) At the end of the day, a bond paying coupons can be thought of as a series of zero coupon bonds, so the theory extends rather easily (except for the fact that you'll need to work with matrices)

Review Questions

The first few questions use periodic compounding. The last one uses continuous compounding.

Q: Calculate the modified duration for a 5-year annual coupon bond assuming the spot rate curve is a flat 5% and the coupons are 6% of face.

A:

Year	CF	PV(CF)	PV(CF * t)
1	60	57.14	57.14
2	60	54.42	108.84
3	60	51.83	155.49
4	60	49.36	197.45
5	1060	830.54	4152.69

PV(CF) = $60 * 1.05^{-1} + 60 * 1.05^{-2} + \ldots + 1060 * 1.05^{-5}$ = 1,043.29
PV(CF * t) = $1 * 60 * 1.05^{-1} + 2 * 60 * 1.05^{-2} + \ldots + 5 * 1060 * 1.05^{-5}$ = 4,671.61

Mac Dur = 4,671.61 / 1,043.29 = 4.48
Mod Duration = 4.48 / 1.05 = 4.265

Q: Using the information above, calculate the expected price of the bond if interest rates move by +0.5%.

A: Current price of the bond = 1,043.29

% Δ Price = -ModDur * Δi $\rightarrow$ -4.265 * 0.005 = -2.13%

New Price = 1043.29 * [1 + (-4.265 * 0.005)] = 1021.05

Note, all of the calculations were done in excel, so there could be some rounding error.

Q: Using the bond above, calculate the key rate duration for year 3. Use the spot curve given below.

T	r_t
1	0.01
2	0.02
3	0.03
4	0.04
5	0.05

A:

Year	CF	PV(CF)
1	60	59.41
2	60	57.67
3	60	54.91
4	60	51.29
5	1060	830.54

Price = 1,053.81

Shocking the 3rd year interest rate by 0.001, you get the following price:

Year	CF	PV(CF)
1	60	59.41
2	60	57.67
3	60	**54.75**
4	60	51.29
5	1060	830.54

Price = $60 * 1.01^{-1} + 60 * 1.02^{-2} + 60 * \mathbf{1.031}^{-3}\ 60 * 1.04^{-4} + 1060 * 1.05^{-5}$ = 1,053.65

KRD(3) = [(1,053.81 - 1,053.65) / 1,053.81] / 0.001 = 0.151

Q: The above process is a lot of work. Find a short cut formula.

A: Think about the bond above as a series of zero coupon bonds with maturities of 1-5 years. We know that the weighted average modified duration of that portfolio of zero coupon bonds will equal the modified duration of the cash flows above.

With that in mind, calculate the "weights" for each zero coupon bond. The weight times the modified duration is equal to the key rate duration.

Year	CF	PV(CF)	Mod Dur of Zero Coupon Bond	Weight	KRD
1	60	59.41	0.9901	0.056	0.056
2	60	57.67	1.9608	0.055	0.107
3	60	54.91	2.9126	0.052	0.152
4	60	51.29	3.8462	0.049	0.187
5	1060	830.54	4.7619	0.788	3.753

Mod Dur of ZCB → PV(CF * t) / [(1+i) * PV(CF)]

Only one cash flow so simplifies into [t / (1+i)]

Since this is periodically compounded, the modified duration of a zero coupon bond is the term divided by (1+i). The Macaulay duration is equal to the term.

Weight → PV_t(CF) / Total PV of Cash flows

Another way to think about this → when you add up the key rate durations you get the modified duration. Using the modified duration formula presented earlier, we can break this into component pieces to solve for the key rate durations.

$$Mod\ Dur = \frac{PV(CF \times t)}{(1+i)PV(CF)}$$

Each component piece is [CF_t * t * $(1+i)^{-t}$] / [(1+i) * PV(CF)]

For year 3 → KRD = [60 * 3 * 1.03^{-3}] / [1.03 * 1053.81] = 0.152

The above formula can be repeated very quickly on exam day!

Q: Using the continuously compounded spot rate curve. Calculate the duration and key rate durations for a 5-year annual coupon bond with coupons of 6% of face.

T	s_t
1	0.01
2	0.02
3	0.03
4	0.04
5	0.05

A:

Duration = PV(CF * t) / PV(CF)

$PV(CF * t) = 1 * 60 * e^{-0.01*1} + 2 * 60 * e^{-0.02*2} + 3 * 60 * e^{-0.03*3} + 4 * 60*e^{-0.04*4} + 5 * 1060*e^{-0.05*5}$
PV(CF * t) = 4,671.36

$PV(CF) = 60 * e^{-0.01*1} + 60 * e^{-0.02*2} + 60 * e^{-0.03*3} + 60*e^{-0.04*4} + 1060*e^{-0.05*5} = 1{,}048.54$

Duration = 4,671.36 / 1,048.54 = 4.455

To find KRD_i --> $PV(CF_i)$ / Total PV(CF) = weights
Zero coupon bond durations = maturity

T	PV(CF)	KRD of Zero Coupon Bond	Weight	KRD
1	59.40	1.0000	0.057	0.057
2	57.65	2.0000	0.055	0.110
3	54.84	3.0000	0.052	0.157
4	51.13	4.0000	0.049	0.195
5	825.53	5.0000	0.787	3.937

They sum to the total duration of 4.455.

Note, for continuously compounded spot rates, modified duration = Macaulay duration.

LRM Formula List

Writing down the formulas on exam day is a key aspect of getting the maximum amount of points. This formula list is a compilation of some of the important formulas on the syllabus. This list is meant to help facilitate memorization and should not be considered to include every formula.

Section A: Risk Identification

Study Note on Parameter Risk (LRM-108-14)

Coefficient of variation (CV)

$$CV(L)^2 = \frac{CV(X)^2}{E(N)} + \frac{VM}{E(N)}$$

Estimated standard deviation of regression analysis

$$\widehat{\sigma_y} = \sqrt{\frac{SSE}{N-2}}$$

Maximize likelihood

$$L(x;\theta) = \prod f(x_i)$$

$$NLL(x;\theta) = -\sum \ln f(x_i)$$

A New Approach for Managing Operational Risk

Cost of Risk

Cost of risk = Expected Loss + Cost of Capital * Unexpected Loss

Poisson Distribution

$$P(X = x|\lambda) = \frac{\lambda^x}{x!}e^{-\lambda}, \;\; x = 0,1,2 \ldots and \;\; \lambda > 0$$

Negative Binomial Distribution

$$P(X = x|r,p) = \frac{\Gamma(r+x)}{x!\,\Gamma(r)}p^r(1-p)^x, \qquad x = 0,1,2,.. \; and \; r > 0, \;\; 0 < p < 1$$

$$\Gamma(r) = (r-1)!$$

The Binomial Distribution

$$P(X = x|p,n) = \frac{n!}{x!\,(n-x)!}p^k(1-p)^{n-k}, \qquad x = 0,1,2 \ldots n \; and \; 0 \le p \le 1$$